SM223 – Calculus III with Optimization Fall 2017
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Review Quiz 2

Instructions. You have 20 minutes to complete this review quiz. You <u>may</u> use your calculator. You may not use any other materials. Put your answers on the separate answer form provided.

1. The tangent vector to the curve $\vec{r}(t) = \langle 2t, \sin t, \cos t\rangle$ at $t = \pi$ is:

   (a) $(2\pi, -\pi, 0)$
   (b) $(2, -1, 0)$
   (c) $(2, 0, 1)$
   (d) $(2\pi, 0, 1)$
   (e) $(2\pi, -1, 0)$

   $\vec{r}\,'(t) = \langle 2, \cos t, -\sin t\rangle$

   tangent vector at $t = \pi$: $\vec{r}\,'(\pi) = \langle 2, -1, 0\rangle$

2. What is $\vec{r}(1)$, given $\vec{r}\,'(t) = t^2\vec{i} + t^3\vec{j}$ and the initial condition $\vec{r}(0) = \vec{i}$?

   (a) $\frac{1}{3}\vec{i} + \frac{1}{4}\vec{j}$
   (b) $\frac{4}{3}\vec{i} + \frac{1}{4}\vec{j}$
   (c) $\frac{2}{3}\vec{i} + \frac{1}{4}\vec{j}$
   (d) $\frac{4}{3}\vec{i} + \frac{3}{4}\vec{j}$
   (e) $\frac{1}{3}\vec{i} + \frac{3}{4}\vec{j}$

   $\vec{r}(t) = \left\langle \frac{t^3}{3} + C_1, \frac{t^4}{4} + C_2 \right\rangle$

   $\vec{r}(0) = \langle C_1, C_2\rangle = \langle 1, 0\rangle \Rightarrow C_1 = 1, \; C_2 = 0$

   $\Rightarrow \vec{r}(1) = \left\langle \frac{1}{3} + 1, \frac{1}{4}\right\rangle = \left\langle \frac{4}{3}, \frac{1}{4}\right\rangle$

3. Which one of the listed vector-valued functions defines a circle?

   (a) $\vec{r}(t) = \langle 3\cos(2t), 3\sin(2t), 4\rangle$ — circle of radius 3, on the plane $z = 4$
   (b) $\vec{r}(t) = \langle 3\cos(2t), 4\sin(2t), 0\rangle$
   (c) $\vec{r}(t) = \langle 3\cos(t), 3\sin(t), 4t\rangle$
   (d) $\vec{r}(t) = \langle 3\cos(t), 4\sin(t), 0\rangle$
   (e) $\vec{r}(t) = \langle 3\cos^2(t), 3\sin^2(t), 4t\rangle$

4. The two lines

$$\vec{r}_1(t) = \langle 1 + 4t, 2 + 5t, 3 + 6t\rangle \quad \text{and} \quad \vec{r}_2(t) = \langle -6 + 7t, -6 + 8t, -6 + 9t\rangle$$

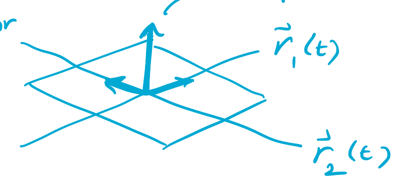
   direction vector $\langle 7, 8, 9\rangle$

   intersect at the point $(1, 2, 3)$. Which one of the listed vectors is perpendicular to the plane that contains both lines?

   direction vector $\langle 4, 5, 6\rangle$

   cross product of direction vectors of both lines

   (a) $\langle 1, 2, 3\rangle \times \langle -6, -6, -6\rangle$
   (b) $\langle 1, 2, 3\rangle \times \langle 7, 8, 9\rangle$
   (c) $\langle 4, 5, 6\rangle \times \langle -6, -6, -6\rangle$
   (d) $\langle 1, 2, 3\rangle \times \langle 4, 5, 6\rangle$
   (e) $\langle 4, 5, 6\rangle \times \langle 7, 8, 9\rangle$



5. Find the length of the curve $\vec{r}(t) = \langle \sin(t), \cos(t), t\sqrt{3}\rangle$ from $t = 0$ to $t = 10$.

   (a) $10 + 50\sqrt{t}$
   (b) $\cos(10) + \sin(10) + 10\sqrt{3}$
   (c) $10 + 10\sqrt{3}$
   (d) 10
   (e) 20

$$\int_0^{10} |\vec{r}\,'(t)|\,dt = \int_0^{10} \sqrt{\cos^2 t + \sin^2 t + 3}\,dt$$

$$= \int_0^{10} \sqrt{4}\,dt = 20$$